

I. Model Problems

To simplify fractional exponents, rewrite the expression as a radical raised to a power. The denominator of the fractional exponent is the root and the numerator is the power.

In other words: $x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Example 1 Write $27^{2/3}$ as a radical and simplify.

$$\begin{aligned} 27^{2/3} &= \left(\sqrt[3]{27}\right)^2 && \text{Rewrite as a radical.} \\ &= 3^2 && \text{Simplify.} \\ &= 9 && \text{Simplify.} \end{aligned}$$

The answer is **9**.

Sometimes you need to write a radical expression using a fractional exponent

Example 2 Write $\sqrt[4]{y^6}$ using a fractional exponent.

$$\begin{aligned} \sqrt[4]{y^6} &= y^{6/4} && \text{Rewrite as a fractional exponent.} \\ &= y^{3/2} && \text{Simplify.} \end{aligned}$$

The answer is $y^{3/2}$.

Sometimes there will be many variables in the radicand. Simplify each variable one at a time, then multiply.

Example 3 Simplify $\sqrt[3]{8x^5y^6z^{11}}$.

$$\sqrt[3]{8x^5y^6z^{11}} = \sqrt[3]{8} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{z^{11}} \quad \text{Rewrite the expression.}$$

$$= 2 \cdot x \sqrt[3]{x^2} \cdot y^2 \cdot z^2 \sqrt[3]{z^2} \quad \text{Simplify.}$$

$$= 2xy^2z^3 \cdot \sqrt[3]{x^2z^2} \quad \text{Simplify.}$$

The answer is $2xy^2z^3 \cdot \sqrt[3]{x^2z^2}$.

Practice Problems are on next page

II. Practice

Evaluate.

1. $25^{1/2}$

2. $1000^{2/3}$

3. $121^{3/2}$

4. $\left(\frac{4}{9}\right)^{-1/2}$

5. $16^{-5/2}$

6. $(-216)^{-1/3}$

7. $\left(\frac{1}{125}\right)^{-1/3}$

8. $49^{-1/2}$

9. $32^{3/5}$

10. $0.04^{1/2}$

Simplify.

11. $\sqrt[3]{a^7}$

12. $\sqrt[5]{z^{11}}$

13. $\sqrt[4]{16c^5d^8}$

14. $\sqrt[3]{27a^3b^5}$

15. $\sqrt[3]{243x^{11}y^8z^{20}}$

16. $\sqrt[3]{8x^5y^6}$

17. $\sqrt[6]{a^8b^8c^{12}}$

18. $\sqrt[3]{40d^5e^6f^4}$

19. $\sqrt[3]{m^{13}n^8p^9}$

20. $\sqrt[3]{64r^{13}s^{15}t^{12}}$

III. Challenge Problems

21. What is the value of $64^{1/12} \cdot 64^{3/12}$?

22. What is the value of $\frac{49^{7/2}}{49^{3/2}}$?

23. Correct the Error

There is an error in the student work shown below:

$$\begin{aligned} \sqrt[3]{24a^3b^4c^7} &= \\ \sqrt[3]{24} \sqrt[3]{a^3} \sqrt[3]{b^4} \sqrt[3]{c^7} &= \\ &= 2ac^2 \sqrt[3]{3bc} \end{aligned}$$

What is the error? Explain how to solve the problem.

24. Circle the expression that is greater:

(a) $(-4)^{23}$ OR $(-4)^3$

(b) $4^{1/2}$ OR $4^{-1/2}$